Coherence and Phase-space I

VSSUP Lectures 2014

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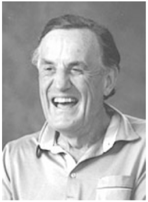
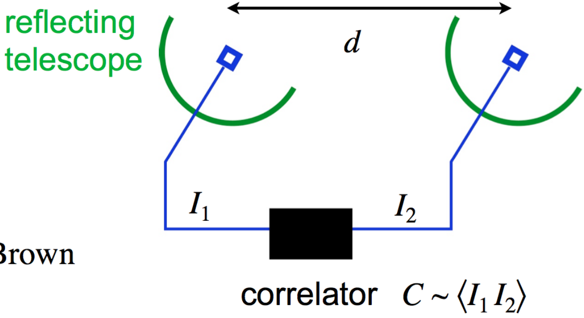
Outline

Starting points - a (partly) Aussie story

Hanbury-Brown used coincidence detection to study stellar diameters, at Jodrell Bank and Narrabri (NSW)



Robert Hanbury Brown
1916-2002



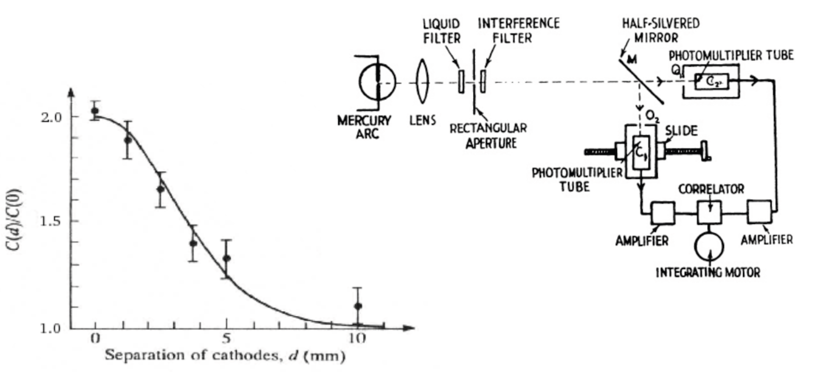
Intensity Interferometry

What is intensity interferometry?

- The current noise in two optical (or radio) telescopes should be correlated for sufficiently small separations d.
- Used to measure distant stellar diameters.
- Short-distance HBT correlation implies photon "bunching".
- Provides evidence for Bose statistics of photons!
- What happens in a BEC? In a Fermi gas?
- When are atoms bunched or antibunched?

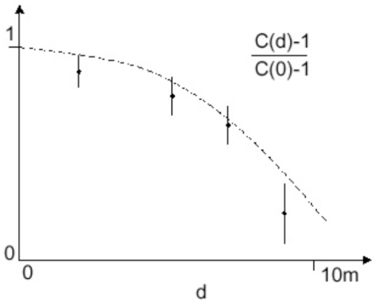
The Hanbury Brown and Twiss experiment 1956

"The experiment shows beyond question that the photons in the two coherent beams of light are correlated and that this correlation is preserved in the process of photoelectric emission."



Measurement of a stellar diameter

Independent photons from different parts of a star "stick together" ("photon bunching")

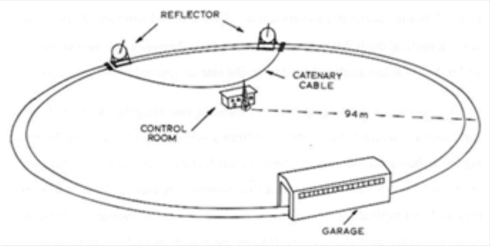


Sirius

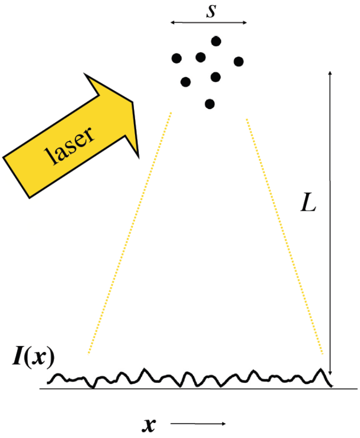
$\theta = 3\times10^{-8}$ radians

Original HBT apparatus

Aerial photo and illustration of original HBT apparatus



Speckle interpretation of intensity interferometry



$l_C = L\lambda/s$

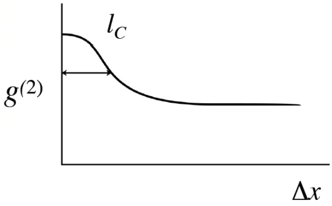
$g^{(2)}(\Delta x) = \langle I(x)\, I(x+\Delta x)\rangle / \langle I\rangle^2$

large $\Delta x \rightarrow$ uncorrelated:

$$\langle I_1\, I_2\rangle = \langle I_1\rangle\langle I_2\rangle$$

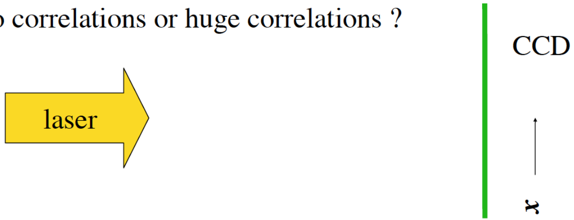
$\Delta x = 0$:

$$\langle I^2\rangle > \langle I\rangle^2$$



Intensity interferometry with a laser

coherence length is very long,
no correlations or huge correlations ?



In practice one does this along the temporal axis.

Experiment (Arecchi, 1966) : $g^{(2)}(t) = 1$

Why? A laser is the closest thing to a stable classical wave

But you do see shot noise $\delta N^2 = \langle N \rangle$

Correlations for a laser on a rotating, ground glass plate

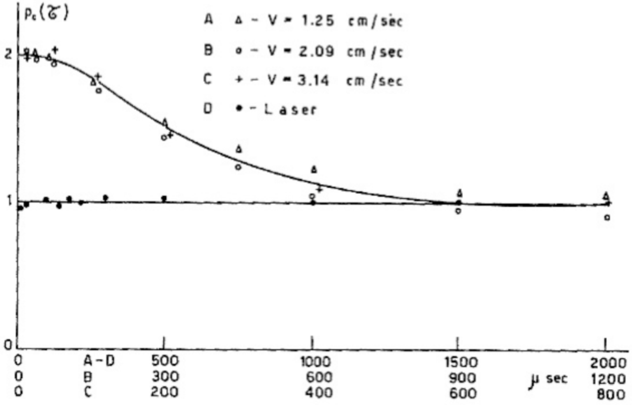
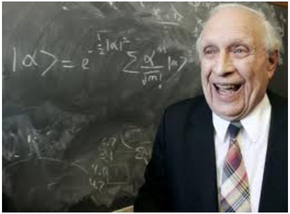


Fig. 1. Conditional probability $p_C(\tau)$ of a second count occurring at a time τ after a first has occurred at time $\tau = 0$.

Photon interpretation using quantized fields (Glauber)



$$\boxed{\begin{aligned} P(t,t') &= \langle : I(t) I(t') : \rangle \\ &= \langle E^-(t) E^-(t') E^+(t') E^+(t) \rangle \end{aligned}}$$

How do we calculate atomic density in quantum mechanics?

For any operator $\hat{O}$, the expectation value of the observable is

$$\left\langle \hat{O} \right\rangle = Tr\left[\hat{\rho}\,\widehat{O}\right]$$

The observable that corresponds to an atomic density at $\mathbf{x}$ is:

$$\hat{n}(\mathbf{x}) = \hat{\Psi}^{\dagger}(\mathbf{x})\,\hat{\Psi}(\mathbf{x})$$

Hence:

$$n(\mathbf{x}) \equiv \langle \hat{n}(\mathbf{x}) \rangle \equiv Tr\left[\hat{\rho}\,\hat{\Psi}^{\dagger}(\mathbf{x})\,\hat{\Psi}(\mathbf{x})\right]$$

Note: Quantum mechanics only tells us about averages, not individual experiments.

How do we calculate atomic correlations?

Suppose we count atoms at multiple locations

The observable that is proportional to the rate of counting atoms simultaneously at $\mathbf{x}_1$ $\mathbf{x}_2$ ... $\mathbf{x}_m$ is:

$$\begin{aligned}\hat{G}^{(m)}(\mathbf{x}_1 \ldots \mathbf{x}_m) &= \hat{\Psi}^{\dagger}(\mathbf{x}_1)\ldots\hat{\Psi}^{\dagger}(\mathbf{x}_m)\hat{\Psi}(\mathbf{x}_m)\ldots\hat{\Psi}(\mathbf{x}_1) \\ &=: \hat{n}(\mathbf{x}_1)\ldots\hat{n}(\mathbf{x}_m):\end{aligned}$$

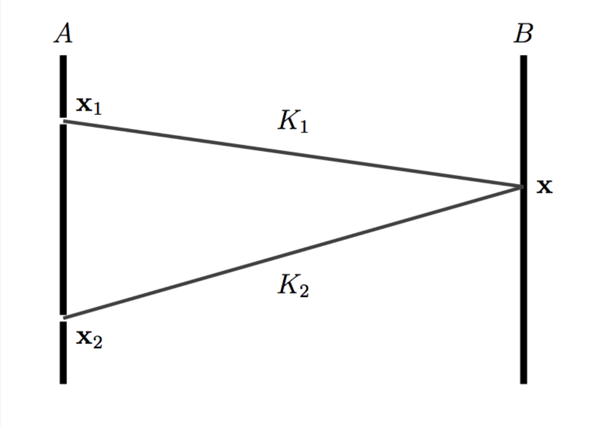
Hence:

$$\begin{aligned}G^{(m)}(\mathbf{x}_1 \ldots \mathbf{x}_m) &= \left\langle \hat{G}^{(m)}(\mathbf{x}_1 \ldots \mathbf{x}_m) \right\rangle \\ &= Tr\left[\hat{\rho}\,\hat{G}^{(m)}(\mathbf{x}_1 \ldots \mathbf{x}_m)\right]\end{aligned}$$

: ... : means normal ordering, annihilation operators to the right

Atomic interference experiments

How can we measure the coherence of atoms?



What about multi-time correlations?

Suppose we count atoms at multiple times and locations, using delayed coincidences

The rate of counting atoms at positions and times: $x_1 = (t_1, \mathbf{x}_1), \ldots\ x_m = (t_m, \mathbf{x}_m)$ is:

$$
\begin{aligned}
G^{(m)}(x_1, \ldots x_{2m}) &= \left\langle \hat{\Psi}^\dagger(x_1) \ldots \hat{\Psi}^\dagger(x_m) \hat{\Psi}(x_{m+1}) \ldots \hat{\Psi}(x_{2m}) \right\rangle \\
&= Tr\left[\hat{\rho}\hat{\Psi}^\dagger(x_1) \ldots \hat{\Psi}^\dagger(x_m) \hat{\Psi}(x_{m+1}) \ldots \hat{\Psi}(x_{2m})\right]
\end{aligned}
$$

Note: two-time correlation functions like $G^{(1)}(x_1, x_2) = \left\langle \hat{\Psi}^\dagger(t_1, \mathbf{x}_1) \hat{\Psi}(t_2, \mathbf{x}_2) \right\rangle$ require momentum transfer, eg Bragg scattering, for their measurement.

m-th order coherence

It is useful to define a normalized coherence function as:

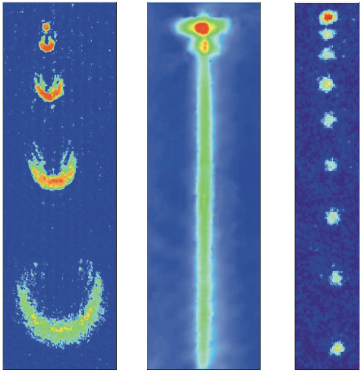
$$g^{(m)}(x_1,\dots x_{2m}) = \frac{G^{(m)}(x_1,\dots x_{2m})}{\sqrt{\prod_{j=1}^{2m} n(x_j)}}$$

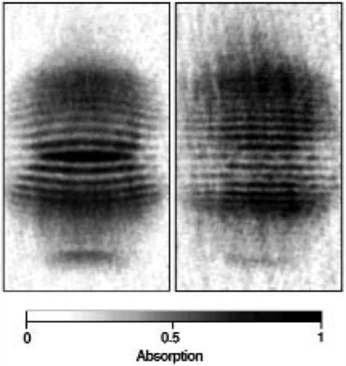
We say we have $m-th$ order coherence if the $m-th$ order correlation factorizes, i.e, we can find a classical field ψ such that:

$$G^{(m)}(x_1,\dots x_{2m}) = \psi^*(x_1)\dots\psi^*(x_m)\,\psi(x_{m+1})\dots\psi(x_{2m})$$

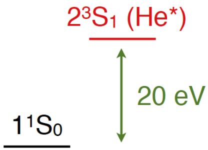
If correlations factorize, field is in a coherent state: $\widehat{\rho} = |\boldsymbol{\alpha}\rangle\langle\boldsymbol{\alpha}|$

Atom Laser Gallery (MIT, Munich, Yale)

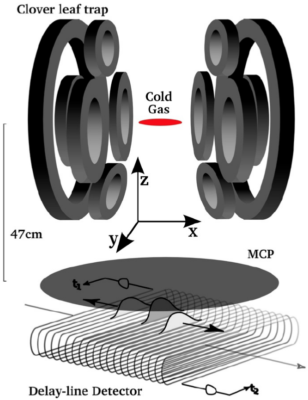




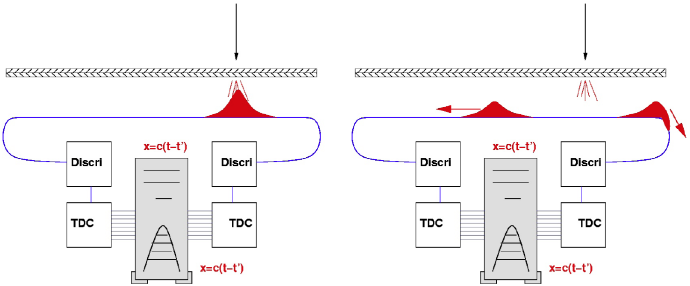
He* apparatus (Westbrook, Aspect: Paris)



- detection by μ-channel plate (He* has 20 eV)
- excellent time (vertical) resolution
- single atom detection 10% quantum eff.
- $\sim$ 500 μm horiz. res. 5*10^4 detectors in //
- $\sim$ 200 ns deadtime

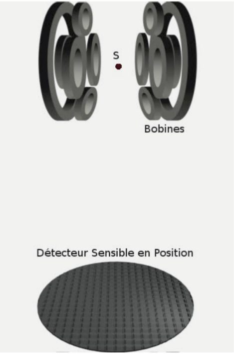


MCP detector with delay line anode



time differences give position information

A "time of flight" observation

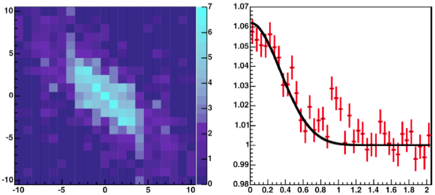


trap

typically 10^5 atoms
time of flight ~ 300 ms
width of TOF ~ 10 ms
we record x,y,t for
every detected atom

detector

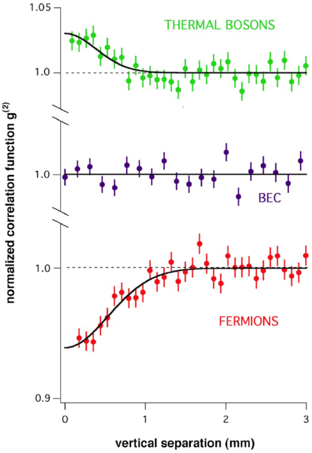
Bose vs Fermi correlations



in detector plane vertical

$g^{(2)}(p - p')$ of a thermal bose gas

M. Schellekens et al. *Science*, **310**, 648 (2005)
T. Jeltes et al. *Nature* **445**, 402 (2007)



What is the quantum state of a BEC?

Like laser theory, suppose a BEC is in a coherent state

$$|\phi\rangle_{CS} = e^{\int d^3\mathbf{x}\left[\phi(\mathbf{x})\hat{\Psi}^{\dagger}(\mathbf{x}) - \frac{1}{2}|\phi(\mathbf{x})|^2\right]}|0\rangle$$

Single mode case: let $\hat{\Psi}(\mathbf{x}) = \hat{a}u(\mathbf{x})$, hence:

$$|\alpha\rangle_{CS} = e^{\alpha\hat{a}^{\dagger} - \frac{1}{2}|\alpha|^2}|0\rangle = e^{-\frac{1}{2}|\alpha|^2}\sum_N \frac{\alpha^N}{\sqrt{N!}}|N\rangle$$

$$G^{(m)}(\mathbf{x}_1 \dots \mathbf{x}_m) = |\alpha u(\mathbf{x}_1)|^2 \dots |\alpha u(\mathbf{x}_m)|^2$$

Problem: can an isolated BEC have a phase?

What about a number state?

Can we instead assume a BEC is in a number state?

$$|\phi\rangle_{NS} = |N\rangle$$

$$G^{(m)}(\mathbf{x}_1 \ldots \mathbf{x}_m) = N(N-1)\ldots(N-m)\left|u(\mathbf{x}_1)\right|^2 \ldots \left|u(\mathbf{x}_m)\right|^2$$

Problem: evaporative cooling is a random process - how can a BEC have zero number fluctuations?

What about a mixture?

The best approximation is a Poissonian mixture of number states

$$\hat{\rho} = e^{-\bar{N}} \sum_N \frac{\bar{N}^N}{N!} |N\rangle \langle N|$$

Or a mixture of coherent states of unknown phase

$$\hat{\rho} = \frac{1}{2\pi} \int d\phi \left|\alpha e^{i\phi}\right\rangle \left\langle \alpha e^{i\phi}\right|$$

Exercise: prove these are one and the same thing!

Glauber P-representation - example of Gaussian phase-space

This is a method for representing the quantum density matrix with normal ordering

$$\widehat{\rho} = \int P(\boldsymbol{\alpha}) \left|\boldsymbol{\alpha}\right\rangle \left\langle\boldsymbol{\alpha}\right| d^{2M}\boldsymbol{\alpha}$$

Phase-space allows positive probabilities for non-squeezed states

- $G^{(m)}(x_1 \ldots x_m) = \left\langle |\alpha|^{2m} \right\rangle |u(x_1)|^2 \ldots |u(x_m)|^2$
- A positive distribution doesn't always exists
- Advantage: Classical-like distribution function!
- Complete coherence $\implies$ $P(\boldsymbol{\alpha}) = \delta(\boldsymbol{\alpha} - \boldsymbol{\alpha}_0)$

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P-distribution for thermal fields

What is the quantum density matrix for a thermal boson field?
Let $E_k = \hbar\omega_k$, $\beta = 1/(k_B T)$, $\hat{n}_k = \hat{a}_k^\dagger \hat{a}_k$, $\mu =$ chemical potential

$$\hat{\rho} = \exp\left[\left(\mu - \beta E_k\right)\hat{n}_k\right]$$

Average boson number: $\bar{n}_k = \left[e^{\beta E_k - \mu} - 1\right]^{-1}$

What is the P-distribution for a thermal field?

$$P(\boldsymbol{\alpha}) = \exp\left[-\left|\alpha_k\right|^2/\bar{n}_k\right]$$

- Correlations: $\left\langle \left(\hat{a}_k^\dagger\right)^m \left(\hat{a}_k\right)^m \right\rangle = \left\langle \left|\alpha_k\right|^{2m} \right\rangle = (m!)\,\bar{n}_k^m$

SUMMARY

Coherence properties of atoms described by correlation functions

Atom counting can be used to obtain correlations!

- Measure fringe visibility to obtain first order coherence
- Higher order coherence is characteristic of a BEC
- Can be measured using nonlinearities and light scattering
- Fermions can have coherence too
- Bosons can be in a phase-mixture of coherent states
- Can think of a BCS state as a coherent state for fermions

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